Data Mining for Climate Model Improvement

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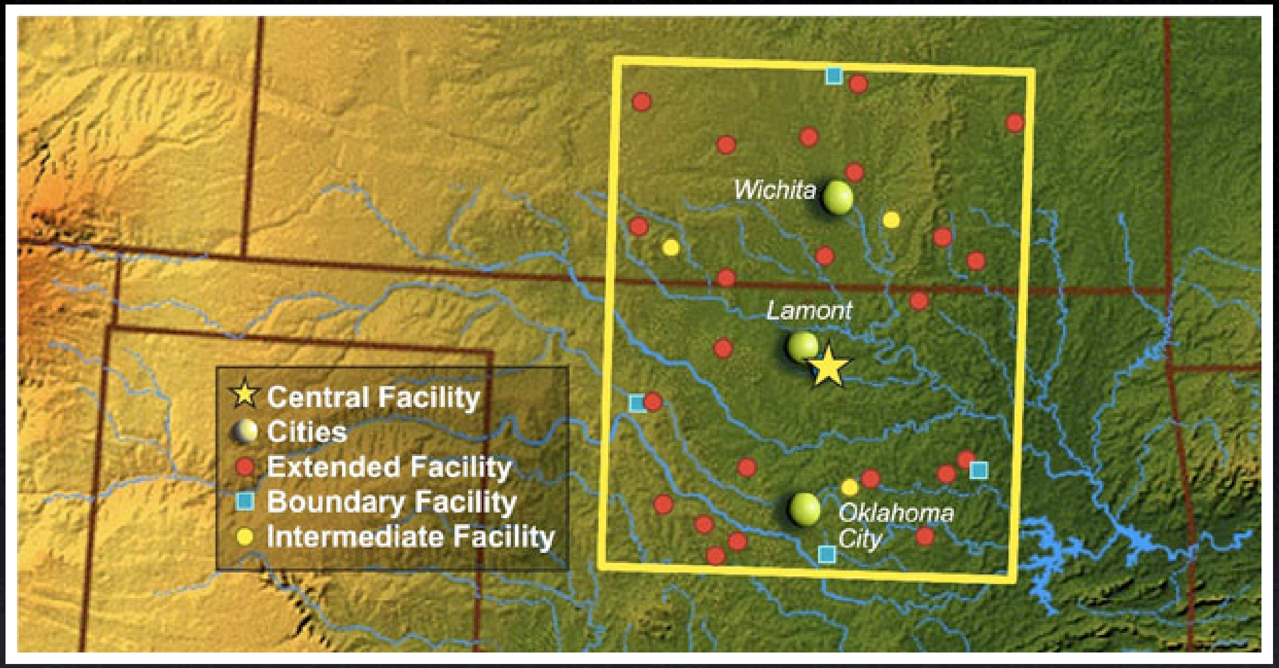
Outline

- Introduction
- Model output and observations
- Estimating multivariate distributions
- Distributional analysis
  - Visual comparisons
  - Hypothesis testing
- Conclusions

Introduction

- Model diagnosis = comparison against observations.
- Model output and observational data sets are too large to make use of.
- Instead, reduce (compress) both sources to multivariate distribution estimates; compare distributions.
- Use tools of statistics and elementary probability to characterize discrepancies.
- Work in progress!

Model Output and Observations



- Study area: Southern Great Plains (SGP) ARM (Atmospheric Radiation Measurement Program) site (north-central Oklahoma).

- Observations: vertical profiles of equivalent potential temperature (θ_e), equivalent saturation potential temperature (θ_{es}) at 35 atmospheric levels, every 30 minutes 1999-2001.

- Model output: GFDL (Geophysical Fluid Dynamics Laboratory's AM2 atmospheric model) vertical profiles of the same variables for the $2.5^\circ \times 2.5^\circ$ grid box containing the SGP site, at the same levels, every 20 minutes 1999-2001.

Model Output and Observations

$\mathbf{x}_{t_1,A}$ = 35 measurements (levels) of θ_e and 35 measurements of θ_{es} at time t_1 for ARM.

$\mathbf{x}_{t_2,G}$ = 35 measurements (levels) of θ_e and 35 measurements of θ_{es} at time t_2 for GFDL.

1:00:30 1:01:00 1:01:30

$\mathbf{x}_{t_{11},A}, \mathbf{x}_{t_{12},A}, \mathbf{x}_{t_{13},A}, \ldots$

Temporal mismatch

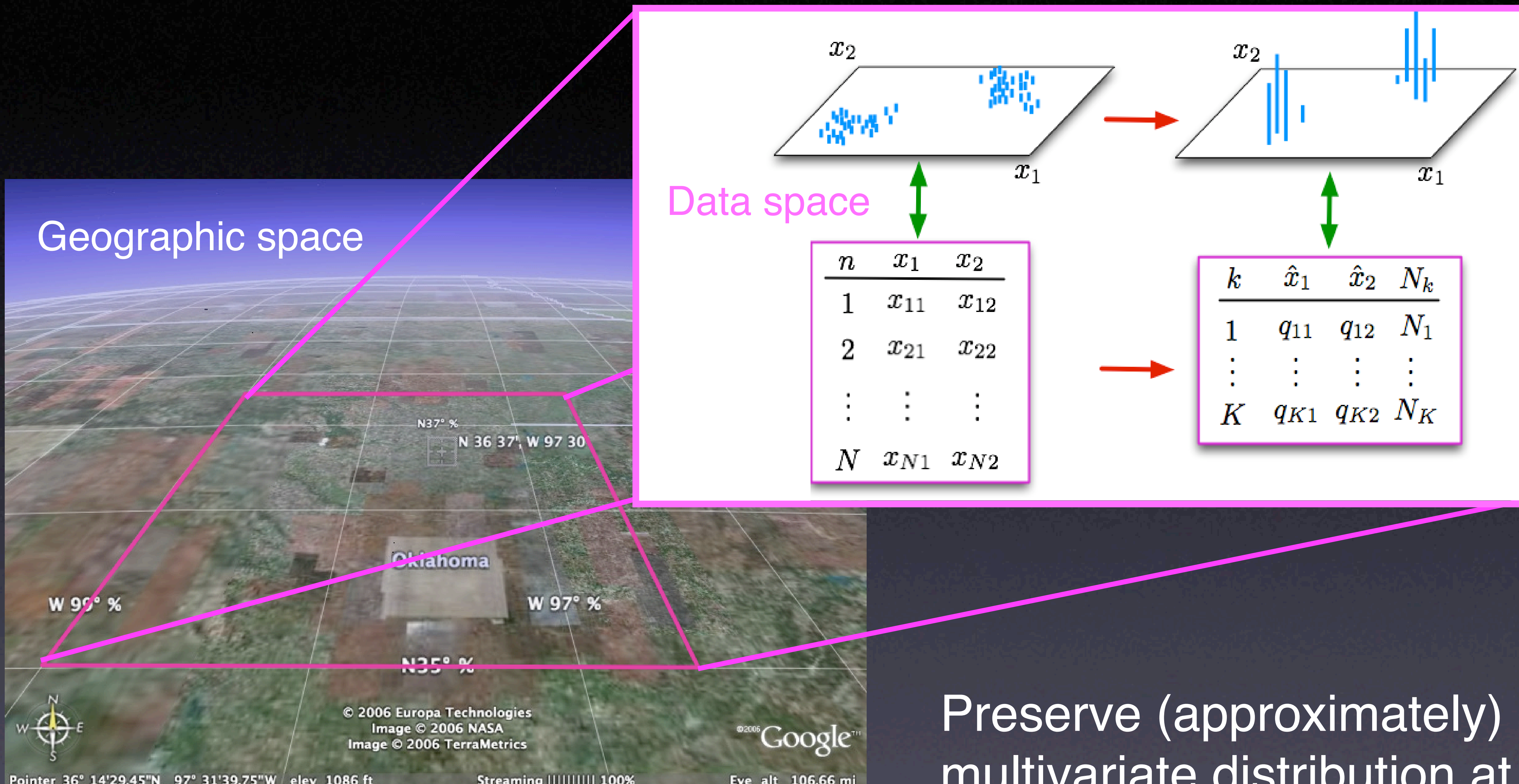
$\mathbf{x}_{t_{21},G}, \mathbf{x}_{t_{22},G}, \mathbf{x}_{t_{23},G}, \ldots$

1:00:20 1:00:40 1:01:00

How to compare?

Interpolate?
Aggregate?
Decimate?

Estimating Multivariate Distributions



Preserve (approximately) multivariate distribution at coarse spatial scale.

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Estimating Multivariate Distributions

- Entropy-constrained vector quantization (ECVQ; Chou, Lookabaugh and Gray, 1989) modified for use as a data summarization algorithm.

- ECVQ can be seen as a clustering algorithm similar to K-means. Different loss function:

$$L = \frac{1}{N}\sum_{n=1}^{N}\left[\|\mathbf{x}_n - y(\mathbf{x}_n)\|^2 + \lambda\left(-\log\frac{N_{y(\mathbf{x}_n)}}{N}\right)\right]$$

$\mathbf{x}_n$ = multivariate data point

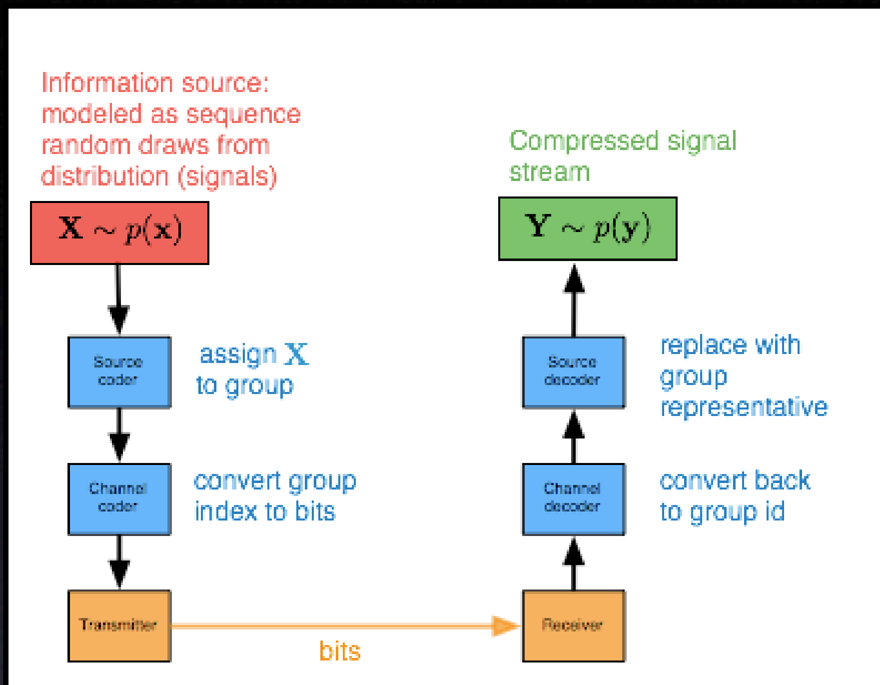
$y(\mathbf{x}_n)$ = centroid of cluster to which data point is assigned

$N_{y(\mathbf{x}_n)}$ = number of data points assigned to cluster with centroid $y(x_n)$

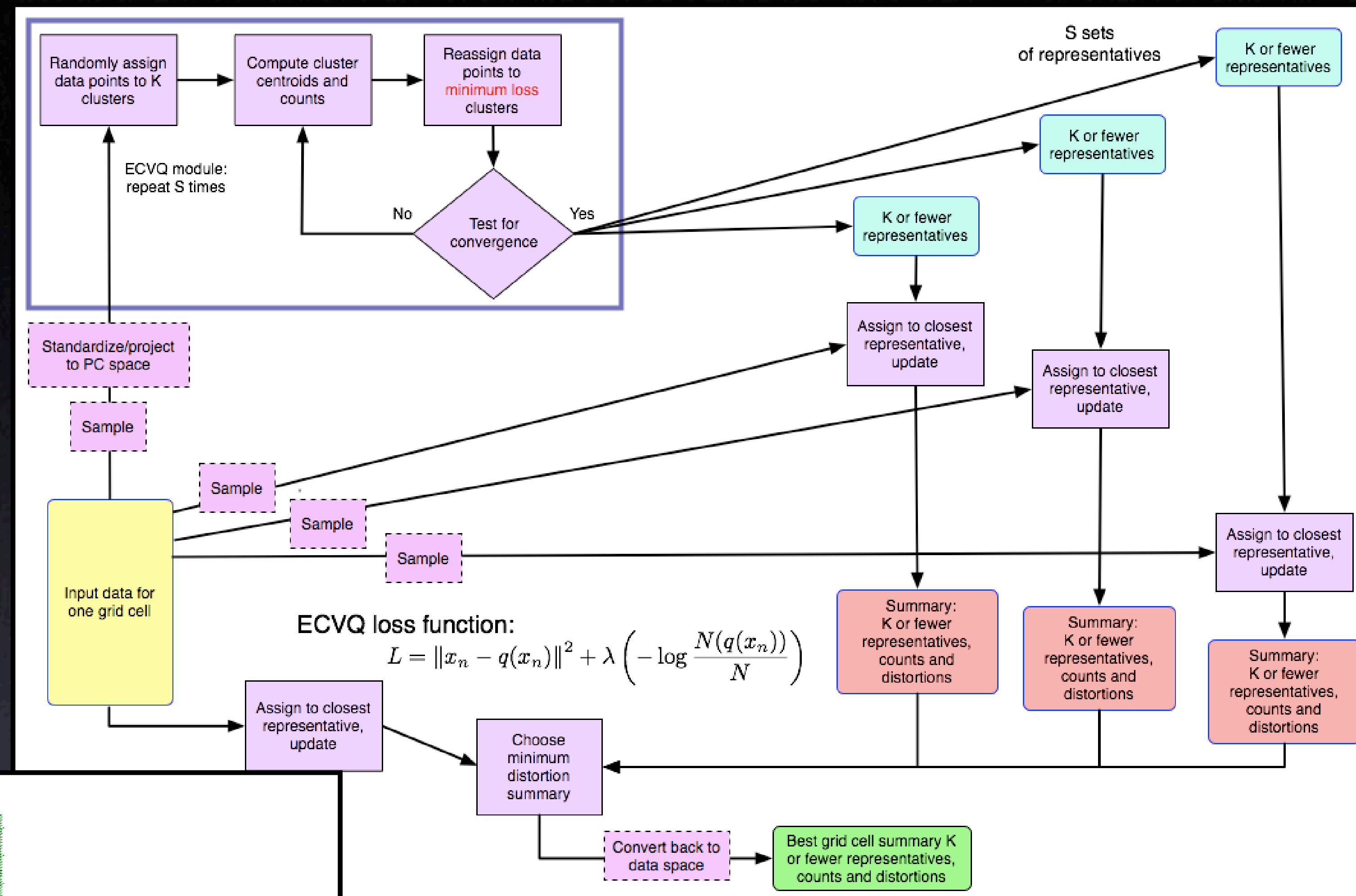
- Result: only as many clusters as necessary to describe the data, up to a maximum of K. (K-means always uses all K clusters.) Information-theoretic complexity of the data determines how many clusters.

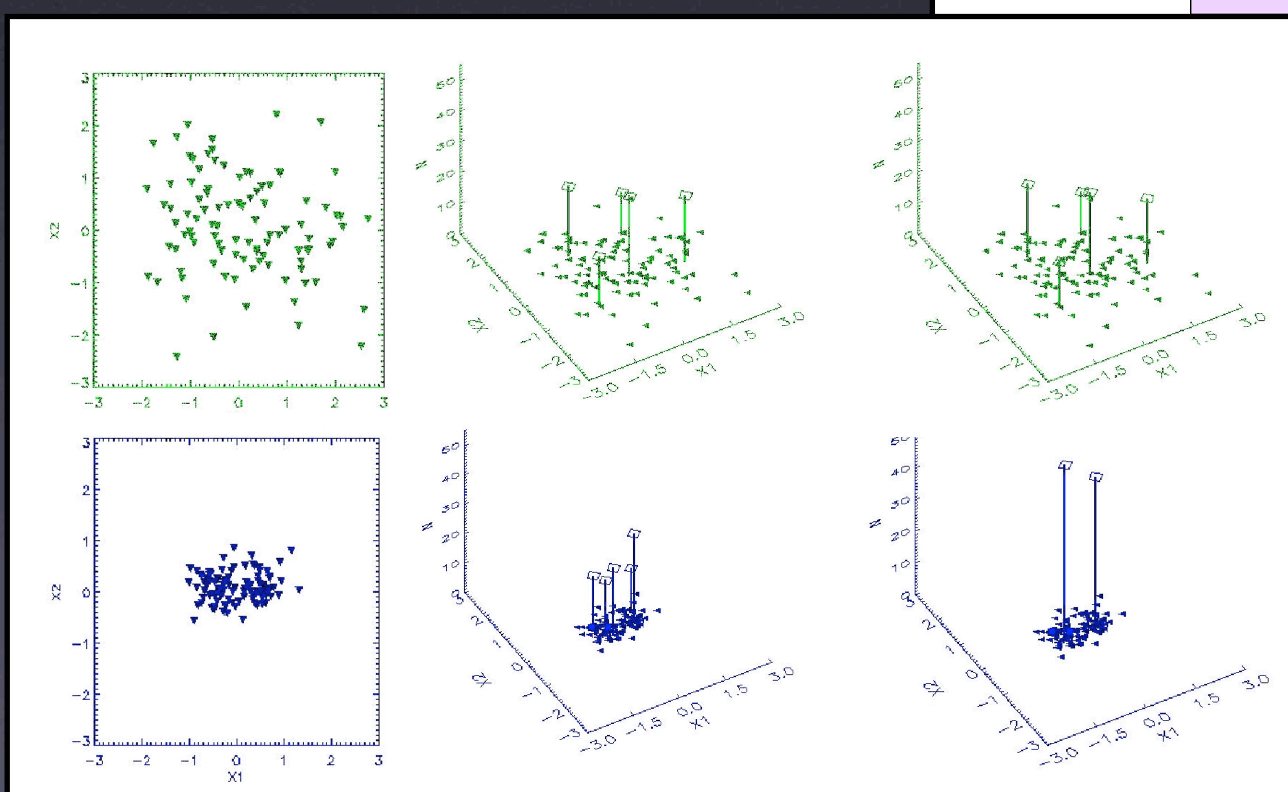
- Strategy: apply ECVQ clustering to data in grid cell(s). Produces a set of cluster centroids and weights for each grid cell.

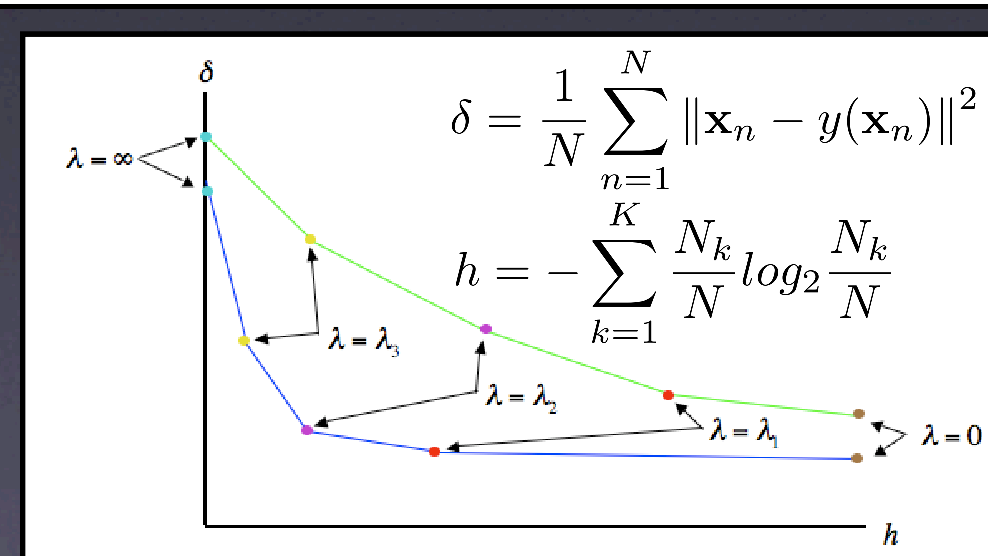
Estimating Multivariate Distributions



Signal processing paradigm







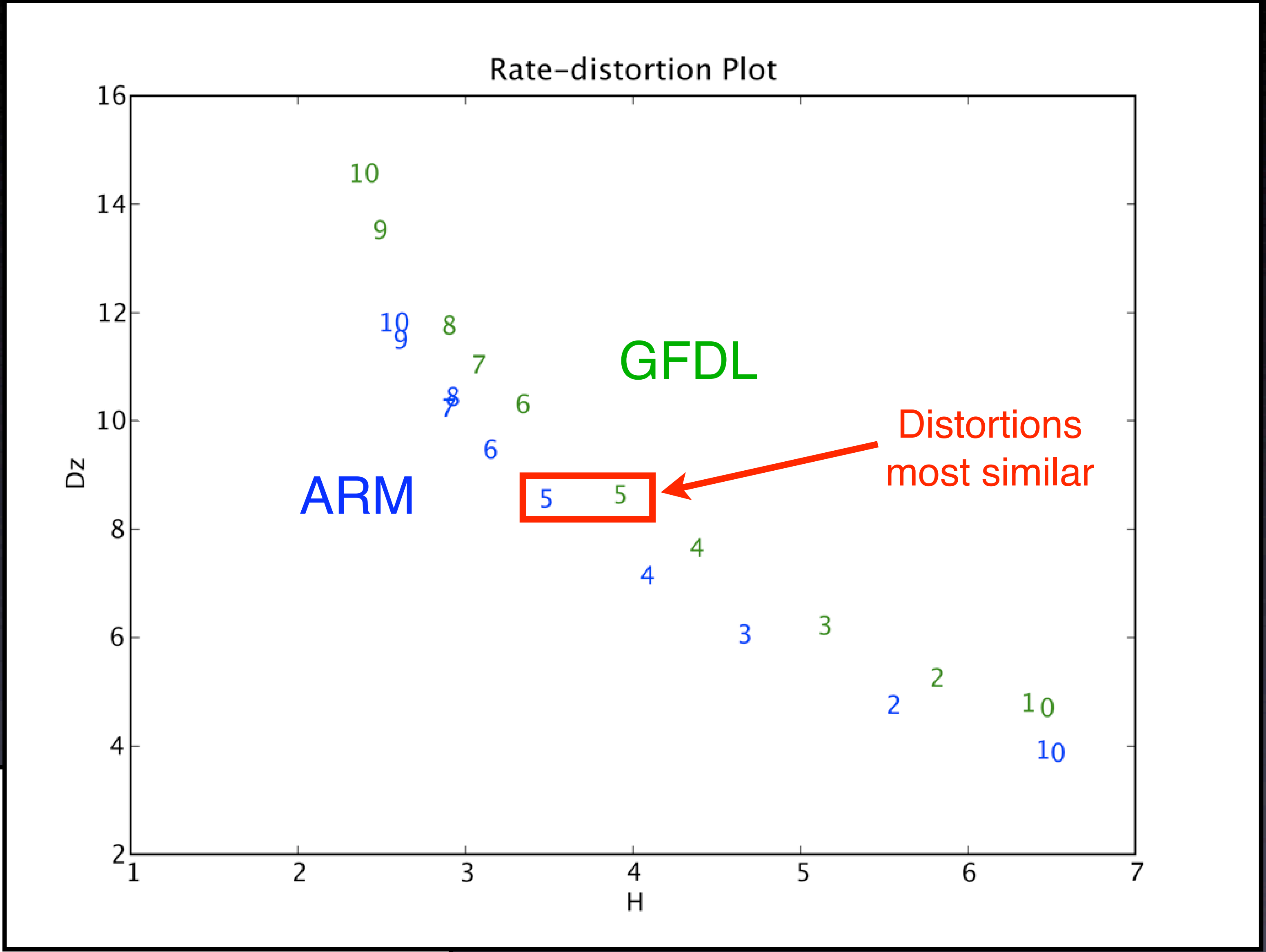
Which λ?

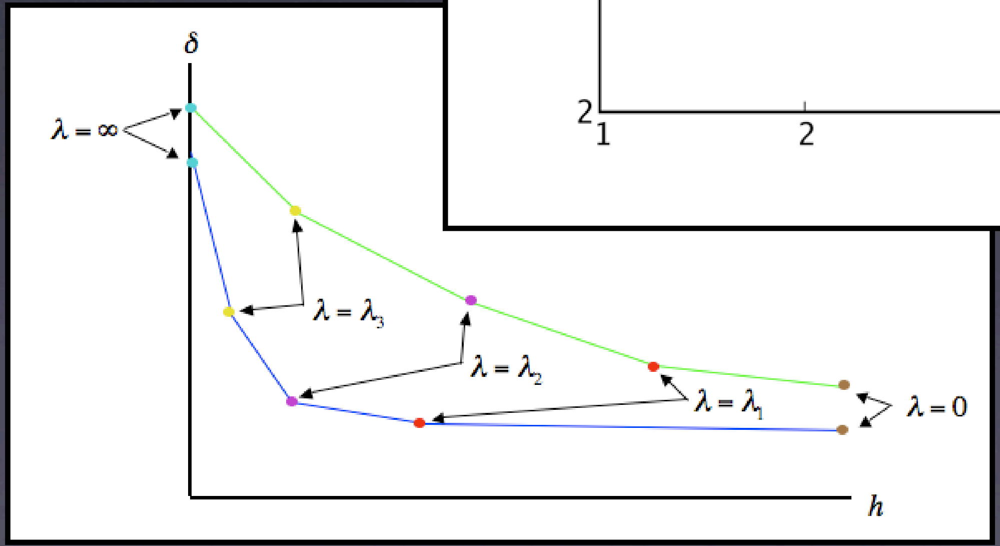
Distributional Analysis
Visual Comparisons

GFDL is more "complex":

Same accuracy requires greater entropy.

Same entropy suffers greater distortion.





Rate-distortion plots for ARM and GFDL.

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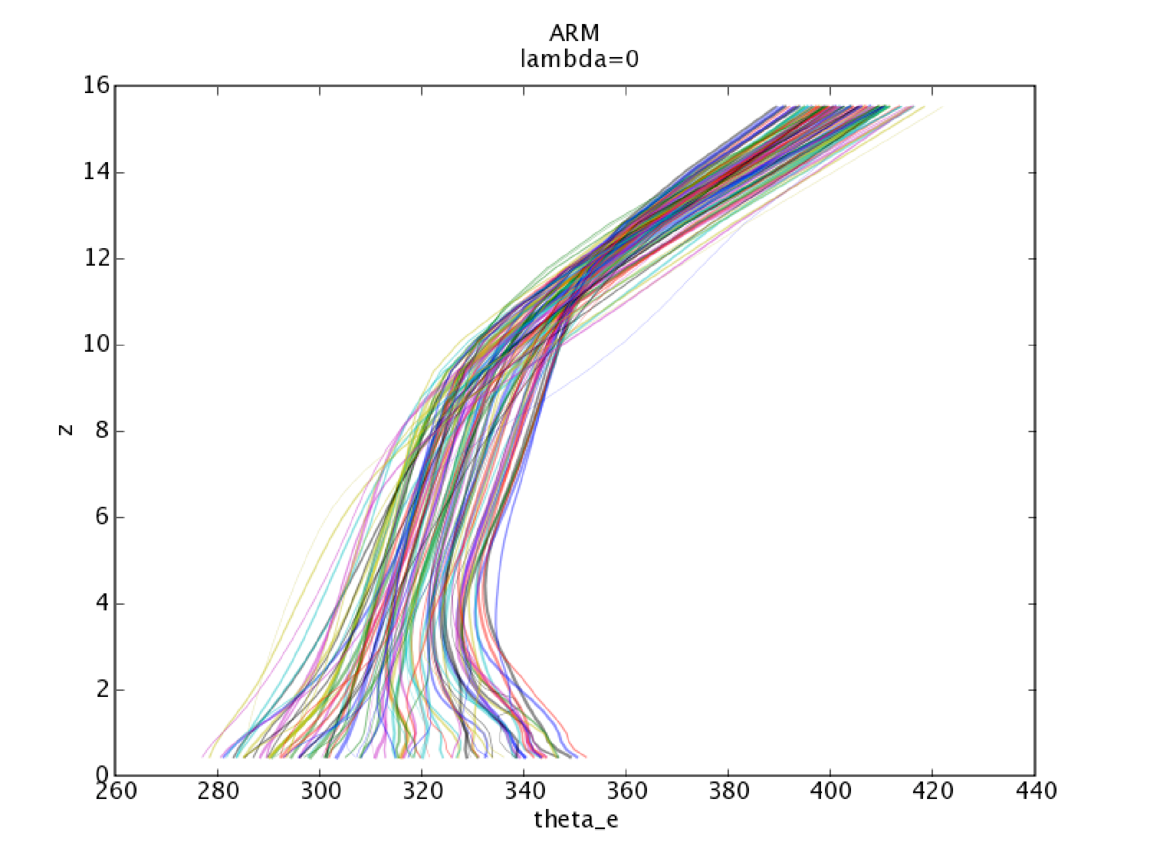
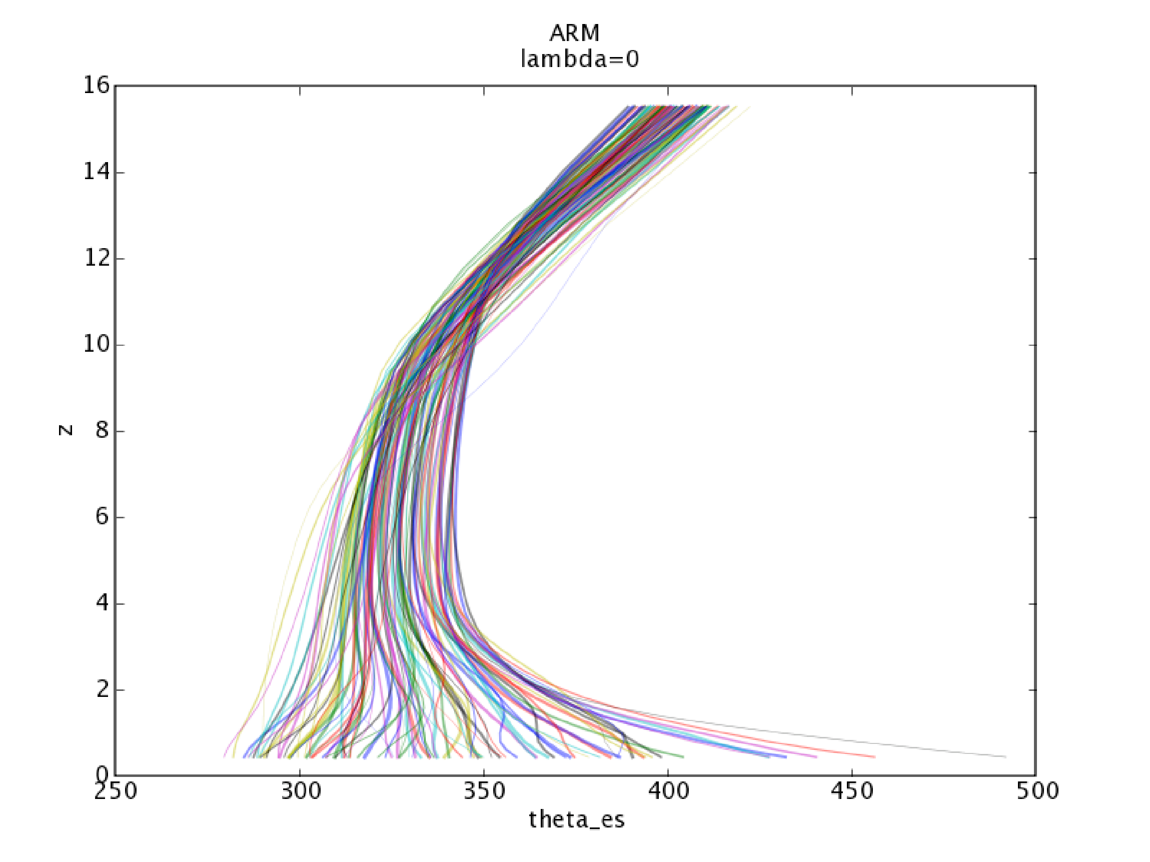
Distributional Analysis
Visual Comparisons





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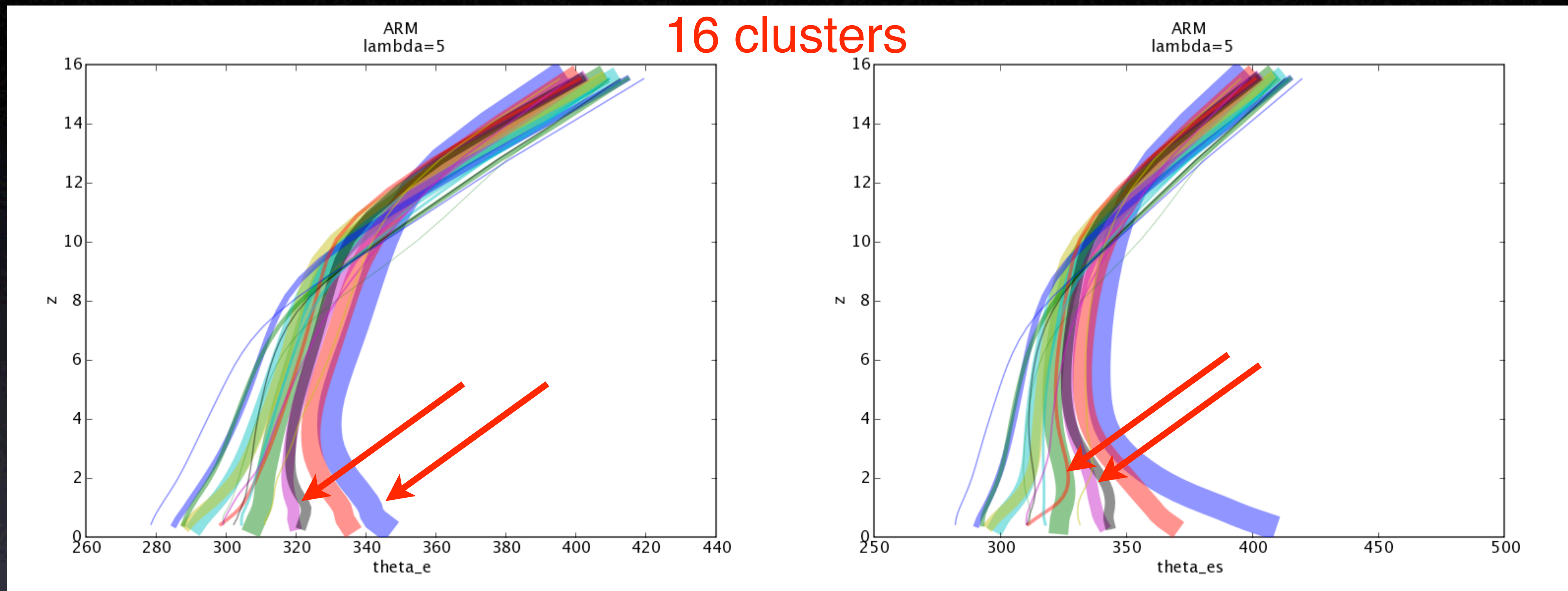
Distributional Analysis
Visual Comparisons

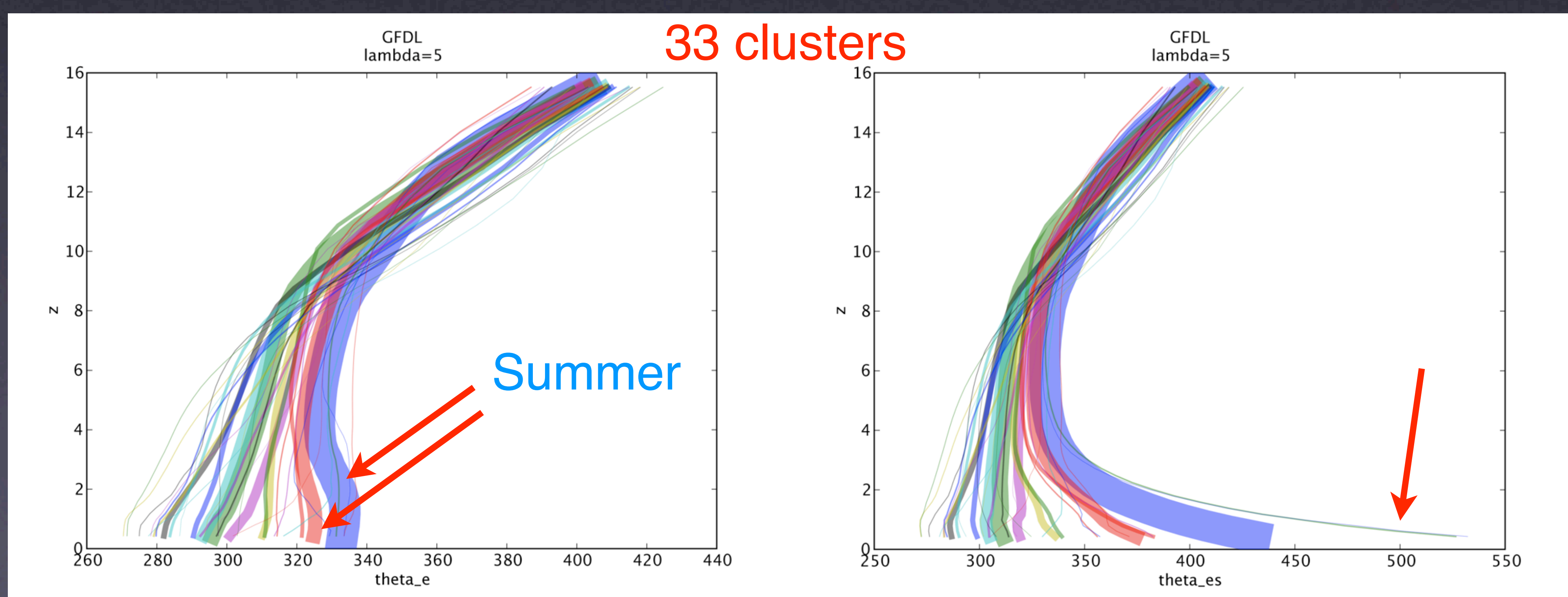
ARM θ_e

ARM θ_{es}



GFDL θ_e

GFDL θ_{es}



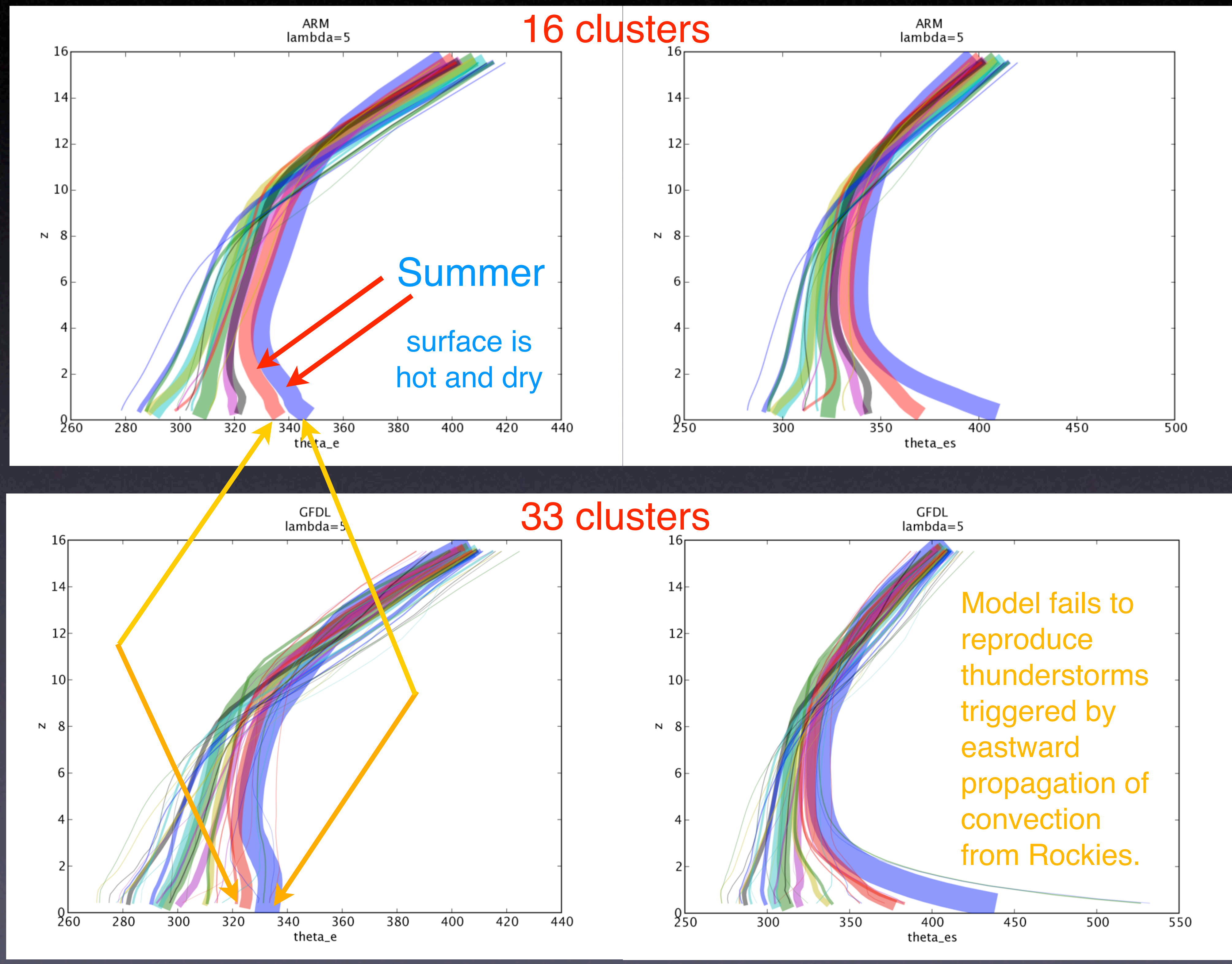
Distributional Analysis
Visual Comparisons

ARM θ_e

ARM θ_{es}

GFDL θ_e

GFDL θ_{es}



Distributional Analysis: Hypothesis Testing

- Are the distributions of ARM and GFDL the “same”?
- Test the hypothesis that the GFDL distribution (P_2) could have been obtained by sampling from a population that looks like the ARM distribution (P_1).
  - Formulate a test statistic that measures the extent to which two distributions differ ($\Delta(P_1, P_2)$).
  - Do the following 100 times:
    - draw N data points randomly from the ARM distribution;
    - cluster them to produce $P_1^*, P_2^*, \ldots, P_{100}^*$;
    - calculate $\Delta_b^* = \Delta(P_1, P_b^*)$, the similarity between P_1 and P_b^*;
    - make a histogram of the Δ_b^* ‘s, $b = 1, 2, \ldots, 100$;
  - If less than 5% of the histogram is greater than the actual $\Delta(P_1, P_2)$, then reject the hypothesis (at the 5% significance level).

Distributional Analysis: Hypothesis Testing

- A distance between distributions:

$$\pi_1 = \{(y_{1k_1}, \pi_{1k_1})\}_{k_1=1}^{K_1} \qquad \pi_2 = \{(y_{2k_2}, \pi_{2k_2})\}_{k_2=1}^{K_2}$$

$$\Delta(\pi_1, \pi_2) = \min_{p_{12}} \sum_{k_1=1}^{K_1} \sum_{k_2=1}^{K_2} \|y_{1k_1} - y_{2k_2}\|^2 p_{12}(y_{1k_1}, y_{2k_2})$$

column constraints

$$\pi_{11} = p_{11} + p_{21} + p_{31}$$
$$\pi_{12} = p_{12} + p_{22} + p_{32}$$
$$\pi_{13} = p_{13} + p_{23} + p_{33}$$
$$\pi_{14} = p_{14} + p_{24} + p_{34}$$

π 's are fixed; fill in p 's such that:

(1) constraints are satisfied
(2) Δ is minimized

$$\pi_{21} = p_{11} + p_{12} + p_{13} + p_{14}$$
$$\pi_{22} = p_{21} + p_{22} + p_{23} + p_{24}$$
$$\pi_{23} = p_{31} + p_{32} + p_{33} + p_{34}$$

row constraints

		y_{11} π_{11}	y_{12} π_{12}	y_{13} π_{13}	y_{14} π_{14}
y_{21}	π_{21}	p_{11}	p_{12}	p_{13}	p_{14}
y_{22}	π_{22}	p_{21}	p_{22}	p_{23}	p_{24}
y_{23}	π_{23}	p_{31}	p_{32}	p_{33}	p_{34}

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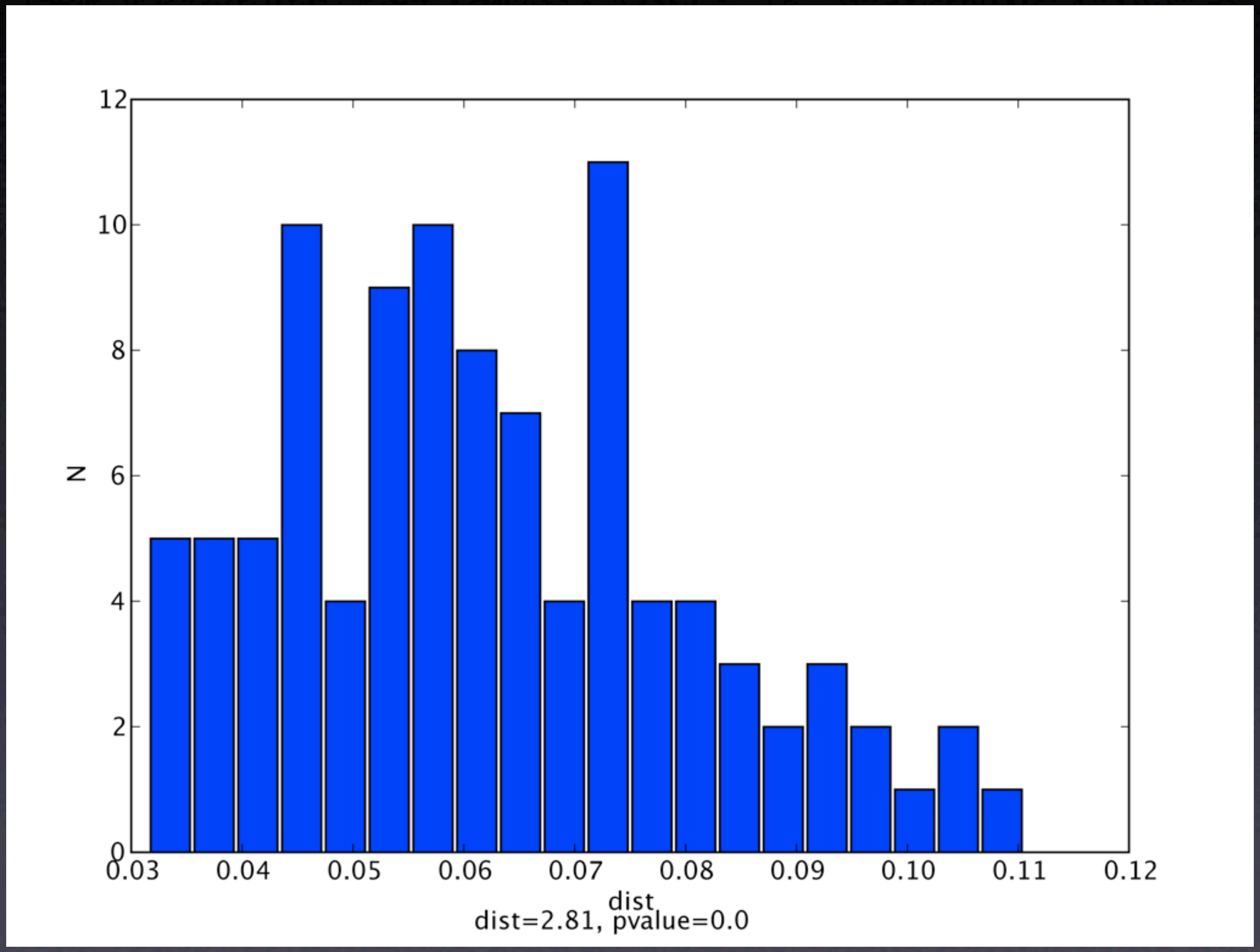
Distributional Analysis: Hypothesis Testing



Histogram of Δ_b^*

Actual $\Delta(P_1, P_2) = 2.81$

Reject the hypothesis; ARM and GFDL distributions are not the same to within sampling variability.

Why?

Which parts of the distribution lead to rejection?

What physical processes do they correspond to?

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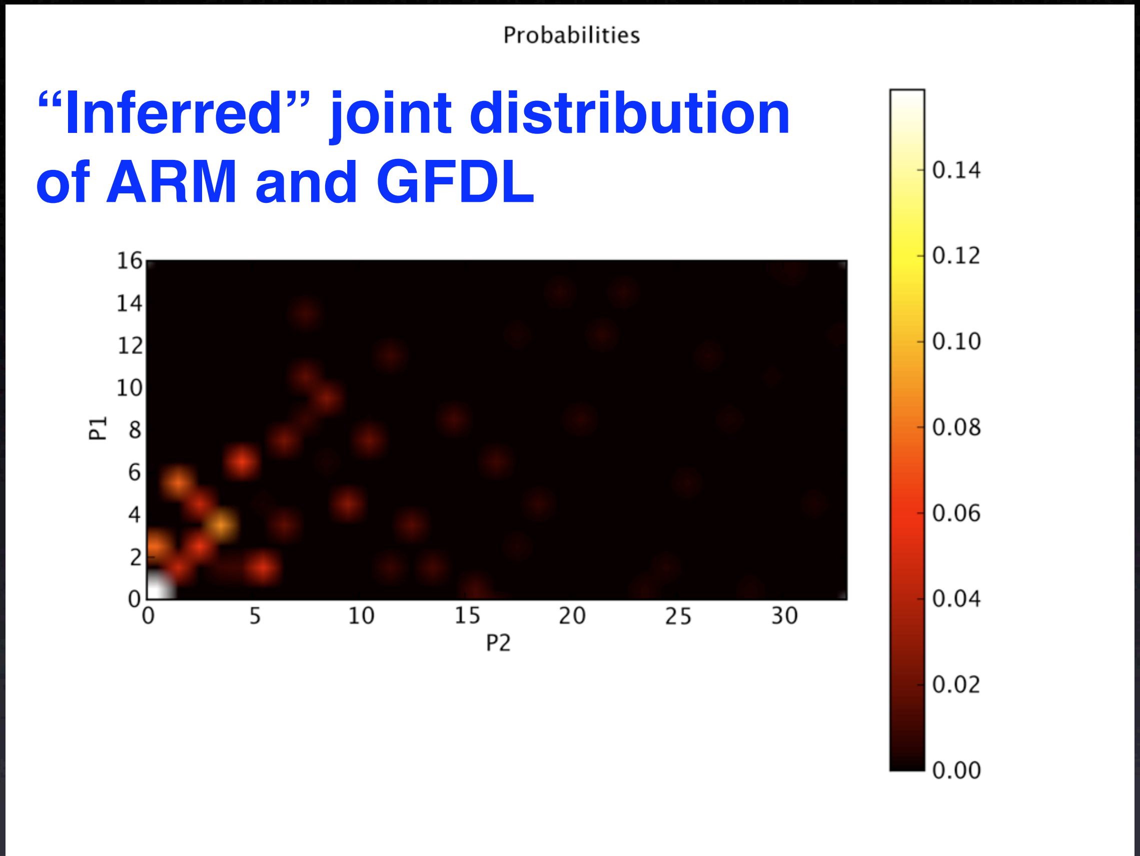
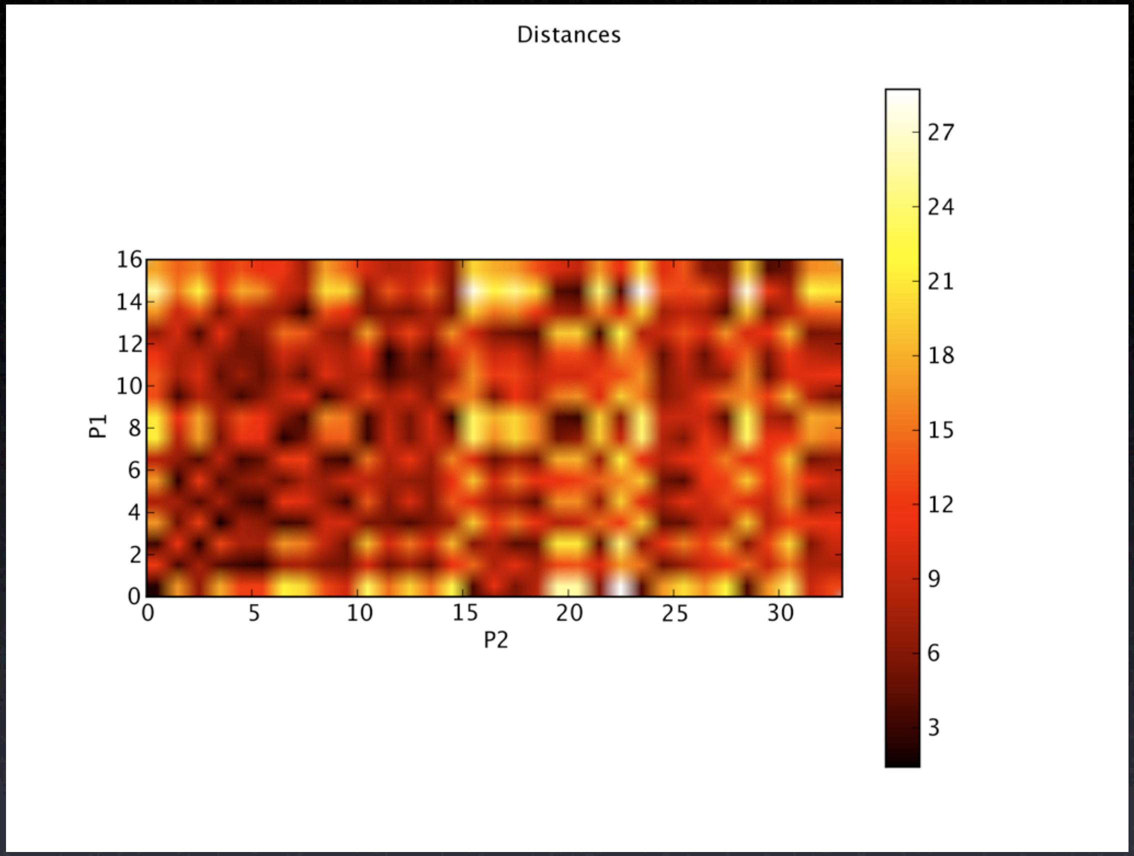
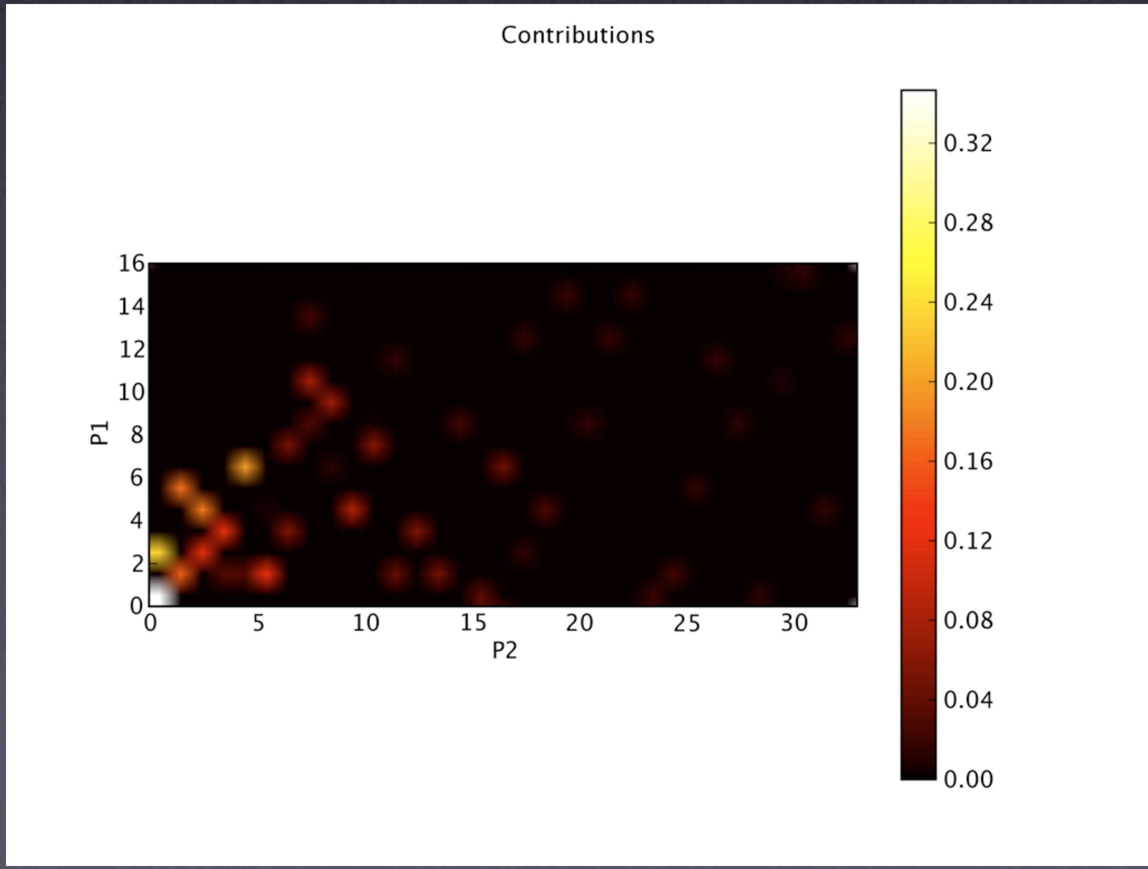
Distributional Analysis: Hypothesis Testing







Largest contributions to $\Delta(P_1, P_2)$ do not correspond to largest distances.

Shows how difficult the problem is!

Distributional Analysis: Hypothesis Testing

An Alternate Approach

- Each cluster represents a distribution of values with mean vector = representative and dispersion = distortion.

- Markov's Inequality bounds the probability of an observation being more distant from the mean than a given amount:

$$P(X > a) \leq \frac{EX}{a}, \quad X = \|\mathbf{X} - y(\mathbf{X})\|^2 \qquad \text{implies}$$

$$P(\|\mathbf{X} - y(\mathbf{X})\|^2 > 20\delta) \leq 0.05$$

Test a set of hypotheses: GFDL cluster j's representative could have been drawn at random from ARM cluster i's distribution...

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Distributional Analysis: Hypothesis Testing

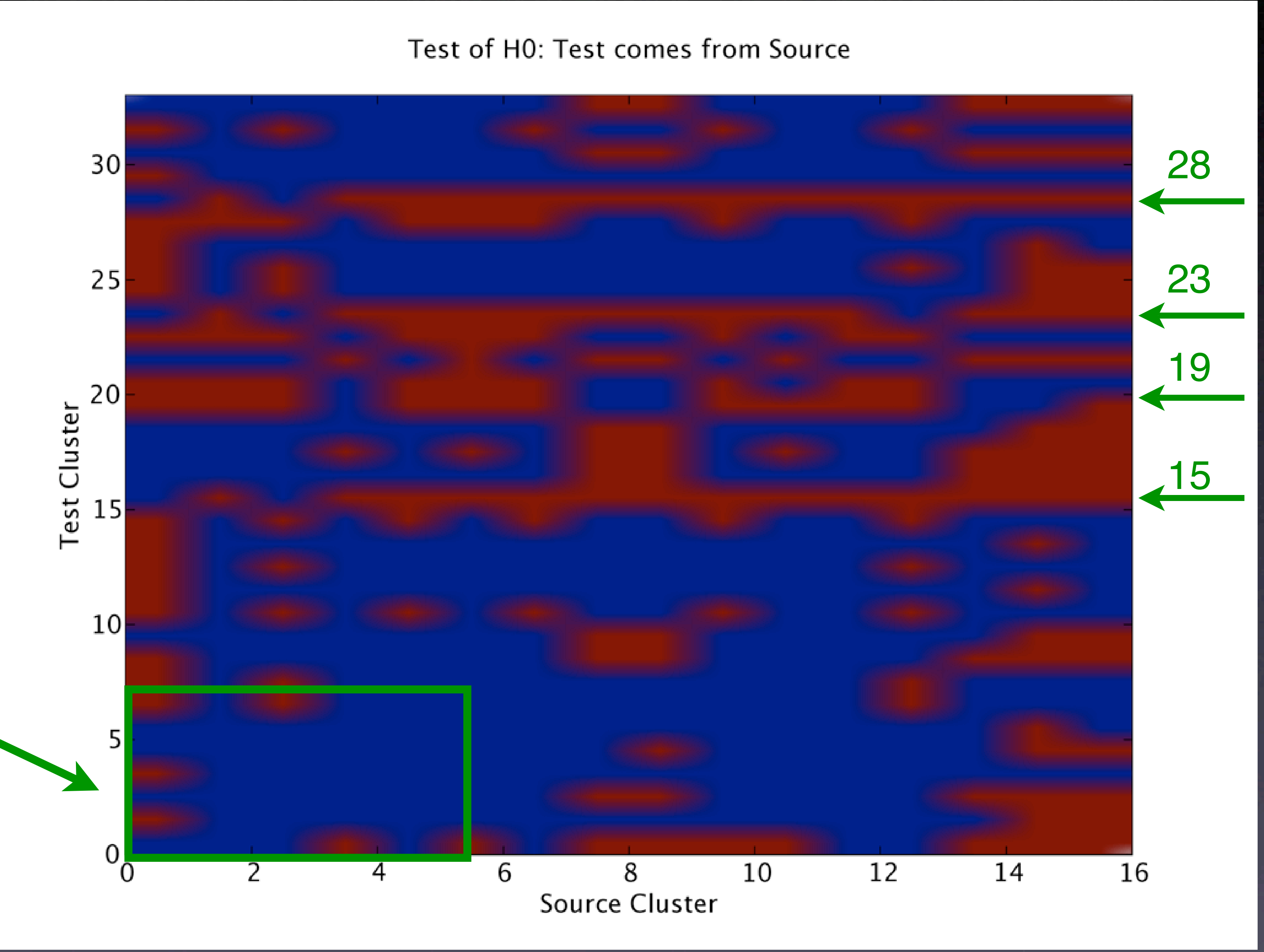
An Alternate Approach

Red=reject

Blue=do not reject

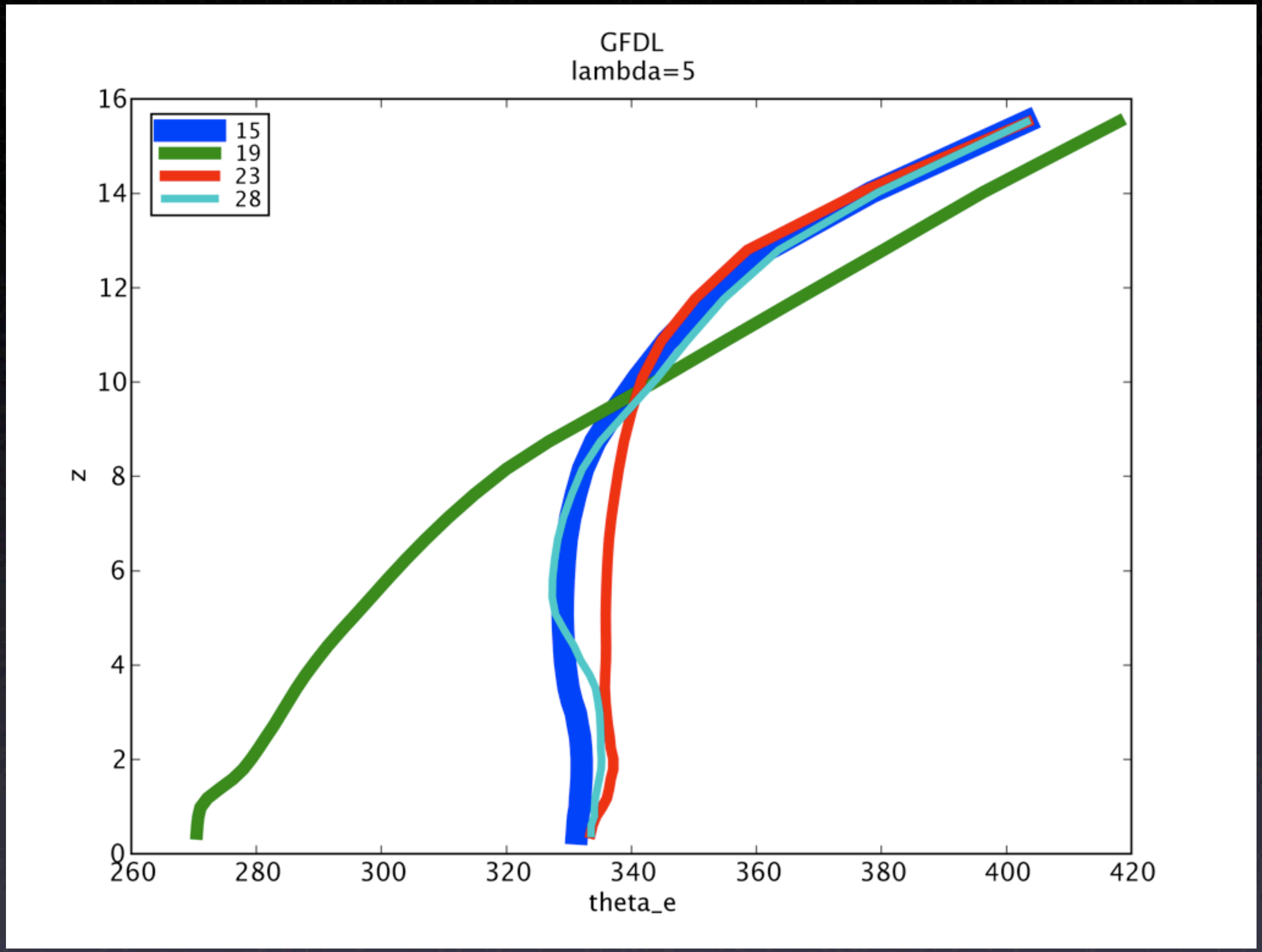
GFDL

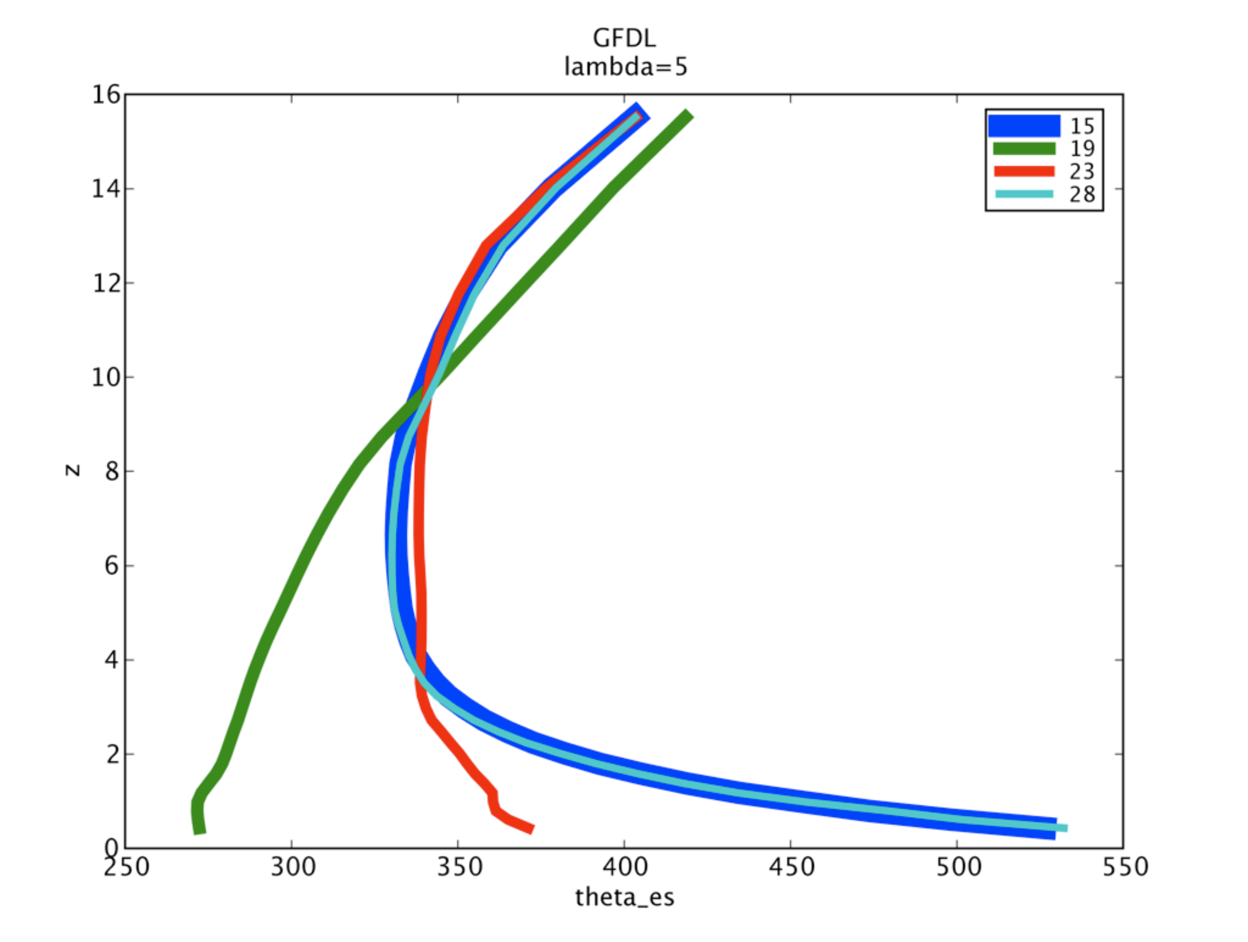
How consistent with the first approach?



ARM

Distributional Analysis: Hypothesis Testing





- GFDL clusters 15 and 28 below 2 km are not physical- too hot and too dry. Precipitation not handled properly.
- GFDL cluster 19: cloudy and unrealistically stable atmosphere.
- GFDL cluster 23?

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Conclusions

- Problem is to discover why model output and comparable data do not agree.
- Estimate discrete multivariate data distributions and compare them to isolate sources of discrepancy.
- Visual inspection is useful, but we need an “autonomous” method suitable for large data sets.
- Two approaches to hypothesis testing using discrete distributions- mixed results, but we are not finished.
- Thanks to ESTO and the AIRS and MISR projects their for support!